

Self-interacting QCD strings and String Balls

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Strings at $T \approx T_c$ are known to be subject to the so-called Hagedorn phenomenon, in which string's entropy and energy cancel each other and result in the evolution of the string into highly excited states, or "string balls". Intrinsic attractive interaction of strings – gravitational for fundamental strings or in the context of holographic models of the AdS/QCD type, or σ exchanges for QCD strings – can significantly modify properties of the string balls. If heavy enough, those start approaching properties of the black holes. We generate self-interacting string balls numerically, in a thermal string lattice model. We found that in a certain range of the interaction coupling constants they morph into a new phase, the "entropy-rich" string balls. These objects can appear in the so-called mixed phase of hadronic matter, produced in heavy ion collisions, as well as possibly in the high multiplicity pp/pA collisions. Among discussed applications are jet quenching in the mixed phase, and also study of angular deformations of the string balls.

I. INTRODUCTION

A. Overview

The first hints for existence of the stringy objects in strong interactions were found in 1960's, way before QCD, quarkonia and their linear potentials between color charges: they came from the Regge phenomenology and the Veneziano scattering amplitude. Theoretical attempts to derive membranes (string worldhistories) starting from the perturbative Feynman diagrams were inconclusive, even though such diagrams become planar, or of "fishnet" kind, in the 't Hooft's large N_c limit. Only with the advent of AdS/CFT correspondence the gauge-string duality became exact for some gauge theories, alas not (yet) for QCD.

Important role of the QCD strings at finite temperatures stems from the fundamental fact that strings, unlike particles, have an exponentially growing density of states [1, 2]. A decade later it has been noticed by Polyakov [3] and Susskind [4] that the so-called Hagedorn phenomenon with strings is at the heart of the (strong first order) deconfinement phase transition in the (pure) gauge theories. As the string entropy and energy are approximately canceling each other, one can get highly excited strings, or *string balls*, which are the subject of this paper.

Historically, studies of the *self-interacting* string balls started in the framework of fundamental string theory: the theoretical questions discussed were related to the understanding of the transition from the string balls to black holes. We briefly remind the main points of that in section IB below.

Highly excited strings populate the so-called "mixed phase" of gluodynamics at $T = T_c$ and provide an energy/entropy density interpolating between the two values $\epsilon_{min}, \epsilon_{max}$ corresponding to hadronic and QGP phases. QCD with dynamical quarks has a crossover transition, in which the "mixed phase" can also be defined as a narrow strip of temperatures in which similar

evolution happens. The so called "resonance gas" models used to describe it are consistent with Hagedorn picture, since hadronic density of states is in good correspondence with those of QCD strings.

If the energy of heavy ion collisions is tuned appropriately, such conditions can occur as the *initial* state of matter produced: it is related to the so-called "softest point" of the equation of state, see e.g. discussion of it 20 years ago [5]. However, in contemporary "mainstream" experiments with heavy ion collisions, at Brookhaven RHIC and CERN LHC colliders, the mixed phase appears as an *intermediate* condition, between the initial QGP stage and the final hadronic one. Still, in the interval between 1/3 and 1/4 of the evolution proper time the matter exists in the mixed phase, and its properties are important to understand. It is also important for certain observables: e.g. it has been proposed [6] that it dominates in the jet quenching, the problem to which we return below. The string balls have been recently discussed in a completely different context, as *initial* states for the high multiplicity pp collisions: we briefly introduce those ideas in Section IC.

The objective of this paper is to study the role of self-interaction of the string balls, specifically for QCD strings at $T \approx T_c$, which is numerically close to the Hagedorn temperature T_H . We will formulate a new lattice model for those, and simulate numerically ensembles of string balls of various sizes, using space-dependent temperature $T(x)$. The main physics issue studied is the dependence of the string balls on the self-interaction coupling. As we found in Section IV B, there are two radical changes at certain values of this coupling: first, a new regime appears, which we call the "entropy rich" regime; second, the ball undergoes a collapse. Applications of those result include a section on jet quenching in the mixed phase V A, in which we point out that current estimates for the jet quenching parameter can be an order of magnitude enhanced, and the section V B in which study shape fluctuations of the "string balls". The paper is summarized in Section VI A, further directions of research are

discussed in Section VIB.

B. From strings to black holes

Historically, the subject of string self-interaction have been first discussed in the context of fundamental strings in critical dimensions (26 for bosonic strings and 10 for superstrings). The string coupling g_s in this case is a function of the vacuum expectation value of the dilaton field, ϕ : $g_s = e^\phi$ for closed strings and $g_s = e^{\phi/2}$ for open strings. The power of g_s in the string amplitude is then given by the Euler characteristic χ of the string world-sheet. As it is well known, the massless modes of closed strings include gravitons: therefore it is a candidate for the theory of quantum gravity. The subject relevant for this work is the transition between the states of massive “string balls” and the ones of black holes.

When any object gets very massive, one expects it to be described classically. Sufficiently massive string balls, if undergo a collapse, should thus become black holes of the classical gravity.

A string ball can be naively generated by a “random walk” process, of M/M_s steps, where $M_s \sim 1/\sqrt{\alpha'}$ is the typical mass of a straight string segment. If so, the string entropy scales as the number of segments

$$S_{ball} \sim M/M_s \quad (1)$$

The Schwarzschild radius of a black hole in d spatial dimensions is

$$R_{BH} \sim (G_N M)^{\frac{1}{(d-2)}} \quad (2)$$

and the Bekenstein entropy

$$S_{BH} \sim \frac{Area}{G_N} \sim \left(\frac{M}{M_s} \right)^{\frac{d-1}{d-2}} \quad (3)$$

Thus their equality $S_{ball} = S_{BH}$ can be reached at some special critical mass only. This happens when the Hawking temperature of the black hole is exactly the string Hagedorn value T_H and the radius is at the string scale. So, at least at such value of the mass a near-critical string ball can be identified – at least thermodynamically – with a black hole.

However, in order to understand how exactly it happens, one should first address the following puzzle. Considering a free string ball (described by the Polyakov’s near-critical random walk), one would estimate its radius to be

$$\frac{R_{ball,r.w.}}{l_s} \sim \sqrt{\frac{M}{M_s}} \quad (4)$$

for any dimension d . This answer does not fit the Schwarzschild radius R_{BH} given above (2).

The important element missing is the self-interaction of the string ball: perhaps, Susskind was the first who

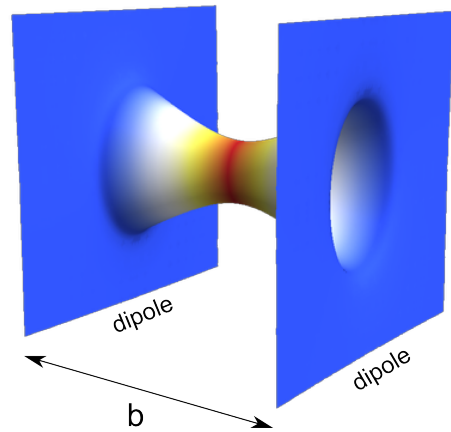


FIG. 1: (Color online) Dipole-dipole scattering due to the “tube” string configuration. The impact parameter \mathbf{b} is the dipole transverse separation.

pointed it out. More quantitative study started by Horowitz and Polchinski [7] had used the mean field approach, and then Damour and Veneziano [8] completed the argument by using the correction to the ball’s mass due to the self-interaction. Their reasoning can be summarized by the following schematic expression for the entropy of a self-interacting string ball of radius R and mass M ,

$$S(M, R) \sim M \left(1 - \frac{1}{R^2} \right) \left(1 - \frac{R^2}{M^2} \right) \left(1 + \frac{g^2 M}{R^{d-2}} \right) \quad (5)$$

where all numerical constants are for brevity suppressed and g is the string self-coupling constant. For a very weak coupling the last term in the last bracket can be ignored and the entropy maximum will be given by the first two terms: this brings us back to the random walk string ball. However, even for a very small g , the importance of the last term depends not on g but on $g^2 M$. So, very massive balls can be influenced by a very weak gravity (what, indeed, happens with planets and stars). If the last term is large compared to 1, the self-interacting string balls become much smaller in size and eventually fit the Schwarzschild radius.

C. String balls emerging in high energy pp scattering

Pomeron description of the high energy hadronic scattering includes production of (two) QCD strings stretched between the receding color dipoles. Zahed and collaborators [9, 10] proposed a semiclassical derivation of the tunneling (Euclidean) stage of the process, based on the so-called “tube” string configuration shown schematically in Fig. 1. Depending on how it is cut, it can be viewed as either a production of two open strings or a closed string exchange between the two color dipoles.

It has been recently pointed out in [11] that the Hagedorn phenomenon and the resulting string balls naturally appear in this setting. The first step is to recognize that the tube geometry of the surface naturally leads to a periodic coordinate and thermal description: the circumference of the “tube” is identified with the Matsubara time $\tau = 1/T$, inverse to the effective string temperature. Furthermore, the tube profile is not uniform along the tube, and, therefore, the temperature T depends on the longitudinal coordinate, that we call σ_W ,

$$T(\sigma_W) = \frac{\chi}{2\pi\mathbf{b}} \frac{1}{\cosh(\chi(\sigma_W - 1/2))}, \quad (6)$$

with its highest value at the center or $T(1/2) \equiv T = \chi/2\pi\mathbf{b}$.

The second step is to note that at certain values of the impact parameter b this temperature corresponds to the Hagedorn value; the effective tension of the string decreases and its high excitations become possible. As a result, as one can expect (and, indeed, sees it directly in the observed elastic scattering profile) that the scattering amplitude for such b exceeds the value interpolated by a Pomeron string expression from large b . One also finds abrupt change to b -independent profile at smaller b , interpreted in [11] as the end of the mixed phase and transition to deconfined (or black hole) phase. Such interpretation suggests prompt production of the “string balls” in a mixed phase, between “cold string” Pomerons at large b and a perturbative domain at small b . High entropy of these balls may lead to very high multiplicity events observed.

It is important to emphasize that the temperature here is just effective description of quantum string excitations in Euclidean partition function under-the-barrier. String balls we discuss in this section therefore appear instantaneously, at $t = 0$, as they emerge in the (Minkowski or real time) part of the system’s path, from the Euclidean tunneling path. In distinction to heavy ion collisions, no time is needed for this “thermalization”.

While in this paper we will discuss QCD strings in $d = 3$ spatial dimension, corresponding to the mixed phase of QCD, we also would like to keep in mind that at time zero the “prompt” near-critical string balls in pp collisions should not be very different. In particular, the lack of one coordinate – the system has near-zero size along the beam direction – is compensated by the presence of an extra holographic direction z , so it is 3-dimensional. (Of course, z is curved and has some end, so it is not exactly the same as another spatial dimension. Also the self-interaction is a bit different. But we do not think that those effects modify the main physics too much.) What is very different is the fate of such string balls: after $t = 0$ they are violently stretched along the beam directions, as the ends of the strings are still attached to the beam particle fragments, moving with a large rapidity.

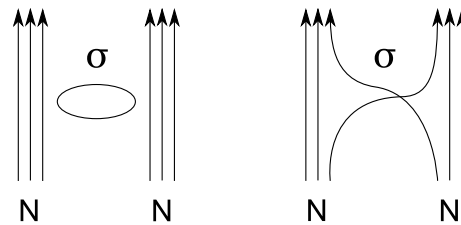


FIG. 2: Color diagrams for σ exchange

II. SELF-INTERACTING STRINGS

A. Self-interaction of QCD strings

For the purpose of this first qualitative study we focus only on the lightest scalar state, known in hadronic phenomenology as the σ meson, or $f_0(500)$ in the PDG13 listings. Its mass $m_\sigma = 0.4 - 0.55 \text{ GeV}$ is comparable to its width $\Gamma_\sigma = 0.4 - 0.7 \text{ GeV}$: that is one of several reasons why this mesonic $J^{PC} = 0^{++}$ resonance has a difficult history, appearing and disappearing in the Particle Data Group tables. The interpretation of its parameters and its dynamical origin has been varying as well, as arguments on this subject had not yet converged. Avoiding the debate we just assume that –perhaps partly – this is the “QCD dilaton”, with the mass $m_\sigma = 0.6 \text{ GeV}$ and zero width, interacting with all hadronic objects with some strength proportional to their $T^{\mu\mu}$.

Only for one particle species – the nucleon N – its coupling is reasonably well known, as it is the main component of the attractive central part of nuclear potential. It takes the Yukawa form

$$V_{NN}(r) = \frac{g_{\sigma NN}^2}{4\pi} \frac{\exp(-m_\sigma r)}{r} \quad (7)$$

and is mostly responsible for the nuclear binding.

(For non-nuclear physicists it may be worth reminding at this point that in NN case it is nearly completely cancelled by the repulsive vector ω exchanges, coupled to the nucleon baryon number. We also remind that this sigma term can be found in phenomenological potentials such as Paris and Bonn ones, or the so-called Walecka model of nuclear forces. More recent treatment uses a more accurate “correlated $\pi\pi$ ” exchange to account for it.

For non-string theorists it may be worth reminding that the fundamental strings and D-branes have also certain charges and repulsive vector forces, canceling attractive ones and making them “BPS-protected”. (Our QCD string is just a bosonic one, without such charges and traces of supersymmetry.)

So, one may think of the string balls we study as some “simplified nuclei”, for which the Fermi blocking, the repulsive vector interactions, and forces related to the spin-isospin interaction are all switched out, with only σ -induced attraction left, binding them together.

There are two color 't Hooft diagrams (see Fig. 2) contributing to this coupling: the first suppressed at the limit of large number of colors, and the second (non-planar one) is in fact suppressed twice, both in the number of colors and flavors. Obviously, only the first one can contribute to the self-interaction of strings. Since we don't know the relative contributions of those diagrams, or, alternatively, the fraction of the "QCD dilaton" in the σ meson, we take the strength of the σNN coupling as an estimate on its *upper* limit. In terms of the "scalar Newton's constant" g_N for the self-interaction of QCD strings its benchmark value is

$$g_N^{max} = \frac{g_{\sigma NN}^2}{4\pi m_N^2} \approx \frac{357}{4\pi} \frac{m_\sigma^2}{m_N^4} \approx 13 \text{ GeV}^{-2} \quad (8)$$

The self-interaction can be studied on the lattice: we were not, however, able to trace papers in which quantitative information on this coupling can be found. So, in what follows we will treat g_N as an unknown parameter.

The simplest problem one can consider analytically is the infinite matter with constant (zeroth order mass) density ρ_0 . The shift in the energy density due to the self-interaction is proportional to the space integral of the potential

$$\frac{|\delta\rho|}{\rho_0} = \rho_0 \int d^3x V(|x|) = \frac{4\pi g_N \rho_0}{m_\sigma^2} \quad (9)$$

Note that the correction diverges for the (gravity-like) massless limit $g_N = \text{const}, m_\sigma \rightarrow 0$, as the static Universe filled with matter cannot exist. However, in the last expression the sigma mass cancels out and, therefore, the result does not depend on its (rather uncertain) value. (This happens because its nuclear parametrization in the form (8) was done with the idea of keeping properties of the nuclear matter independent on it as well).

The previous expression naturally leads to a concept of the critical mass density, at which the negative self-interaction energy cancels the original zeroth order mass, $\delta\rho + \rho_0 = 0$,

$$\rho_c = \left(\frac{m_N^4}{357} \approx 0.28 \frac{\text{GeV}}{\text{fm}^3} \right) \frac{g_N^{max}}{g_N} \quad (10)$$

For the maximal coupling $g_N = g_N^{max}$ it is about twice the mass density for the symmetric nuclear matter $\rho_{n.m.} \approx 0.149 \text{ GeV}/\text{fm}^3$. However, the energy density of the mixed phase of interest is in fact up to an order of magnitude higher: this suggests that the coupling g_N should in fact be substantially smaller than g_N^{max} .

At the density $\rho > \rho_c$, it becomes energetically more favorable to produce new string segments. The process of production stabilizes at the upper high energy density cutoff of our model. This is a scenario preceding a gravitational collapse. On the other hand, we consider our strings to be in a contact with a heat bath of a certain temperature: therefore their stability depends on a (much stronger) condition, based on the *free* energy rather than the energy itself.

Of course, these arguments only apply to very large systems, much larger than the correlation length m_σ^{-1} , and below we will study finite size string balls. We solved some spherically symmetric examples as well, but the results are not particularly instructive to be discussed. Our main objective is to study string configurations of an arbitrary shape, which is a suitable task for the numerical simulations.

Can a collapsing ball be stabilized? One natural cut-off for the strings density follows from the self-avoiding rule in our lattice model. One may also wonder, since σ is a meson, if its effective Lagrangian includes repulsive nonlinear terms $O(\sigma^4)$, on top of its kinetic and mass terms leading to the Yukawa expression used. We have not studied this option, partly because in the AdS/QCD setting – which we describe shortly – the nonlinear actions are well defined and known, yet the gravitational collapse is unavoidable.

The intriguing feature of a collapsing ball is a continuous production of entropy, resulting from the fact that very dense string balls have a huge number of (classical) configurations. So, we have our version of the information paradox. Like evaporating Hawking black holes finally disappear, the string balls at the mixed phase all eventually decay into hadrons, as the heavy ion collision reaches its hadronic phase. The string entropy stored in these balls should also be eventually released into the final clusters.

B. Self-interaction of holographic strings

Although we do not really discuss holographic AdS/QCD string balls in this work, let us still comment on those. Two relevant to us massless fields in the bulk Lagrangian are the *dilaton* and the *graviton*. They are interacting with the stress tensor, $T^{\mu\mu}$ and $T^{\mu\nu}$, respectively, in a standard manner.

Furthermore, the specific dynamics of a high energy collision leads to the near-vanishing values of time and longitudinal (beam) coordinates $x^0, x^1 \approx 0$, so only the "transverse" coordinates $x^2, x^3, x^5 = z$ are left for string fluctuations. As a result, the effective space is also 3-dimensional, as for the usual QCD strings in space. The difference comes from the metric curved in z -direction.

Existence of the confining wall in holographic direction leads to the quantization of the motion in this direction, effectively making propagation in other directions massive. For a specific choices of the wall – e.g. the so called "soft wall" [14] – one can easily calculate the mass spectrum of hadrons: typically one gets linear Regge trajectories. In this sense, massless bulk dilation and graviton correspond to a whole trajectory of massive hadrons in the gauge theory.

Even though we are not ready to discuss subsequent evolution of the string ball at $t > 0$, the gravitational language of the holography allows us to introduce the notion of the "trapped surface". It can be calculated

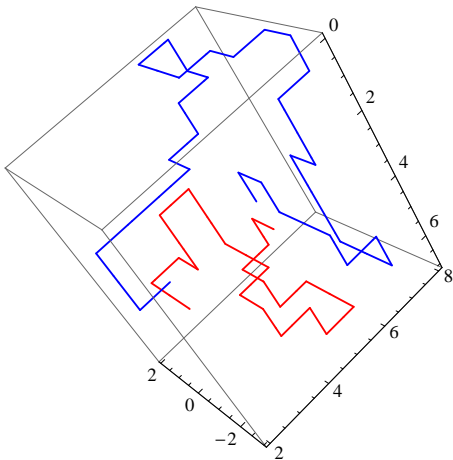


FIG. 3: (Color online) Example of a two-string configuration (a sparse string ball): two strings are plotted as blue and red.

instantaneously, at $t = 0$. It may or may not exist for a given matter distribution: for example, in holographic collisions with a nonzero impact parameter there is a critical value b_c above which the trapped surface disappears [15, 16].

III. THERMAL STRINGS ON THE LATTICE

The strings are placed on links of a ($d = 3$)-dimensional lattice and are in contact with a heat bath. Instead of using boxes, with or without periodic boundary conditions, as is customary in the lattice gauge theory and many other statistical applications, we opted for an infinite space (no box), making instead the temperature T to be *space dependent*. We think it better corresponds to experimental situation. Furthermore, string ball surface is automatically near criticality and thus strongly fluctuating: this aspect will be important for our application of initial deformations below.

The “physical units” in gluodynamics, as in lattice tradition, are set by putting the string tension to its value in the real world

$$\sigma = (0.42 \text{ GeV})^2 \quad (11)$$

Numerical lattice simulations have shown that gluodynamics with $N_c > 2$ has first order deconfinement phase transition, with $T_c/\sqrt{\sigma}$ very weakly dependent on N_c . Numerically, critical temperature of the gluodynamics is $T_c \approx 270 \text{ MeV}$.

It has been further shown that the effective string tension of the free energy $\sigma_F(T)$ decreases with T : a point where it vanishes is known as the Hagedorn point. Since this point is above T_c , some attempts have been made [12] to get closer to it by “superheating” the hadronic phase, yet some amount of extrapolation is still needed.

The resulting value was found to be

$$\frac{T_H}{T_c} = 1.11 \quad (12)$$

The nature of the lattice model we use is very different from that of the lattice gauge theory (LGT). First of all, we do not want to study quantum strings and generate 2-d surfaces in the Matsubara $R^d S^1$ space, restricting ourselves to the thermodynamics of strings in d spatial dimensions.

The lattice spacing a in LGT is a technical cutoff, which at the end of the calculation is expected to be extrapolated to zero, reaching the so-called continuum limit. In our case a is a physical parameter characterizing QCD strings: its value is selected from the requirement that it determines the correct density of a state. Since we postulate that the string can go to any of $2d - 1$ directions from each point (going backward on itself is prohibited), we have $(2d - 1)^{L/a}$ possible strings of length L . Our partition function is given by

$$Z \sim \int dL \exp \left[\frac{L}{a} \ln(2d - 1) - \frac{\sigma L}{T} \right], \quad (13)$$

and hence the Hagedorn divergence happens at

$$T_H = \frac{\sigma a}{\ln(2d - 1)}. \quad (14)$$

Setting $T_H = 0.30 \text{ GeV}$, according to the lattice data mentioned above and the string tension, we fix the 3-dimensional spacing to be

$$a_3 = 2.73 \text{ GeV}^{-1} \approx 0.54 \text{ fm}. \quad (15)$$

It is, therefore, a much more coarse lattice, compared to the ones usually used in LGT.

If no external charges are involved, the excitations are closed strings. At low T one may expect to excite only the smallest ones. With “no self-crossing” rule we apply, that would be an elementary plaquette with 4 links. Its mass,

$$E_{\text{plaquette}} = 4\sigma a \approx 1.9 \text{ GeV}, \quad (16)$$

is amusingly in the ballpark of the lowest glueball masses the of quantum theory. (The lowest “meson” is one link or 0.5 GeV , and the lowest “baryon” is three links – 1.5 GeV with a junction.)

At temperatures below and not close to T_H one finds extremely dilute $\mathcal{O}(e^{-10})$ gas of glueballs, or straight initial strings we put in. Only close to T_H multiple string states get excited, the strings rapidly grow and start occupying larger and larger fraction of the available space.

Before we show the results of the simulation, let us discuss the opposite “dense” limit of our model. We do not allow strings to overlap: the minimal distance between them is one link length, or again about 0.5 fm . Is it large enough for string to be considered well separated? We

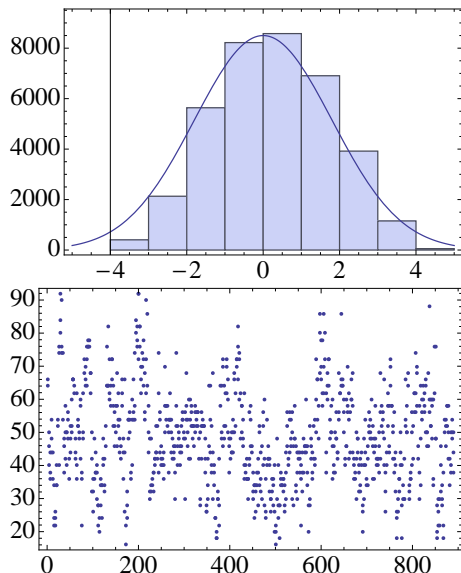


FIG. 4: Upper plot: distribution of all points through all the ensembles along one of the spatial coordinates (in units of a), compared to a Gaussian distribution. Lower plot: dependence of the string length L (a units) on the computer time t (in units of 10 full iterations). Both for the zero self-coupling and $T_0 = 1$ GeV, $s_T = 2a \approx 1$ fm simulation.

think so, as it is about three times the string radius (see discussion below around (29)).

The most compact (volume-filling or Hamiltonian) string wrapping visits each site of the lattice. If the string is closed, then the number of occupied links is the same as the number of occupied sites. Since in $d = 3$ each site is shared among 8 neighboring cubes, there is effectively only one occupied link per unit cube, and this wrapping produces the maximal energy density,

$$\frac{\epsilon_{max}}{T_c^4} = \frac{\sigma a}{a^3 T_c^4} \approx 4.4 \quad (17)$$

(we normalized it to a power of T_c , the highest temperature of the hadronic phase). It is instructive to compare it to the energy density of the gluonic plasma, for which we use the free Stefan-Boltzmann value

$$\frac{\epsilon_{gluons}}{T^4} = (N_c^2 - 1) \frac{\pi^2}{15} \approx 5.26 \quad (18)$$

and conclude that our model's maximal energy density is comparable to the physical maximal energy density of the mixed phase we would like to study.

One remaining issue is treatment of color number. In practice we ignore it, considering thermal excitations of two strings we always initiate the system with. We also think of those strings as direct and reverse color fluxes from two neutral hadrons, which appear in hadronic collisions: it basically mean that all our strings have all only *one* and the same color. Their mutual repulsion – or no-crossing rule – is in this case natural. All we simulate

is the Hagedorn phenomenon due to exponentially large number of string states, ignoring pre factors due to the N_c .

Some justification for that comes from the fact that (apart from the properties of the deconfined phase itself) very little N_c dependence is seen in the lattice gluodynamics data, for a review see [13]. One may however still wonder if one should assign specific colors to strings in the model and account for the fact that two overlapping flux tubes with *different* colors may be partially allowed. In this first study we simply did not want to make our model too complex.

IV. NUMERICAL SIMULATIONS

A. String ball without a self-interaction

Our algorithm consists of a sequence of updates for the each string segment, such that the configuration gradually approaches equilibrium. In order to thermalize the string and to generate a statistical ensemble, we use the following three types of elementary updates:

$$\begin{array}{ccc} \text{---} & \rightarrow & \text{---} \\ | & & | \\ \text{---} & & \text{---} \end{array} \quad (19)$$

$$\begin{array}{ccc} \text{---} & \leftrightarrow & \text{---} \\ | & & | \\ \text{---} & & \text{---} \end{array} \quad (20)$$

$$\begin{array}{ccc} \text{---} & \rightarrow & \text{---} \\ | & & | \\ \text{---} & & \text{---} \end{array} \quad (21)$$

Where the new “corners” and “staples” are chosen in a way avoiding self-intersections. A new configuration is then accepted with the probability from the heat bath (Metropolis) algorithm,

$$P_A = \min \left[1, \exp \left(\frac{E_{old} - E_{new}}{T} \right) \right], \quad (22)$$

where (E_{old}) E_{new} is the total energy of the (old) new configuration, and T is the temperature in the region of space, where the update is performed. We introduce a space-dependent temperature with a Gaussian profile

$$T(r) = T_0 \exp \left(-\frac{r^2}{2s_T^2} \right) \quad (23)$$

As the self-interaction is absent ($g_N = 0$), the physics is simple: in the “cold” regions of space $T(x) < T_H$ the string's entropy times temperature is less than its energy and the string segments are only present if they should cross the region in order to connect fixed string ends. In the “hot” region, where $T(x) > T_H$ the string gets strongly excited.

Since in hadronic collisions the color flux conservation requires production of an even number of strings, (most)

of our simulations are initialized by the two-string configurations. The endpoints are separated by a fixed distance $3a \sim 1.5$ fm and are not moved by the update algorithm.

In Fig. 4 we show an example of history of such simulations, as the string length versus the computer time t/t_m . The time is in units $t_m = 10$ of the entire string update cycles. The total run (equilibration time excluded) is typically about $(1 - 2) \times 10^4$ iterations. The necessary run length actually was found to be dependent on the ball size: the example we will now use corresponds to a “medium-size ball” with a length of about 50 links and a mass of about 25 GeV.

The integral distribution over all three coordinates is close to the Gaussian one, as is exemplified in the upper figure. Yet it is not just a Gaussian ensemble of random points, as the points constitute extended objects - strings. One can see in the lower part of Fig. 4 that while the average properties had apparently converged, the (computer time) history of the system displays rather large fluctuations. The reason for those is the near-critical conditions at the ball surface, where the string has effectively a very small effective tension. Furthermore, if one looks at the individual configurations – e.g. those displayed in Fig. 3 – one can see that, in spite of relatively heavy string balls, most of the space remains unoccupied.

As the parameter s_T of the ball size is reduced, the mean length (and thus the ball’s mass) is strongly diminished as well. Two examples of the length distribution shown in Fig. 5 make this point clear. While at $T_0 = 1$ GeV, $s_T = 1.5a$ (dark blue histogram) one finds a

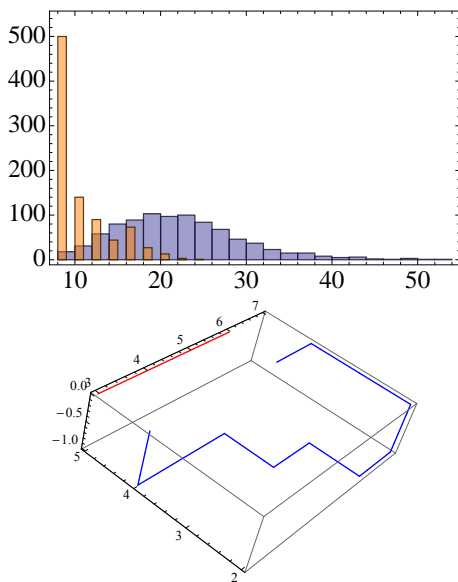


FIG. 5: (Color online) Upper plot: distribution over the string lengths (in units of a) in our simulations. Dark (blue) histogram is for $T_0 = 1$ GeV, $s_T = 1.5a$, the light (orange) one is for $T_0 = 1$ GeV, $s_T = 1.0a$. The lower plot shows a typical configuration in the second ensemble, with only one string excited.

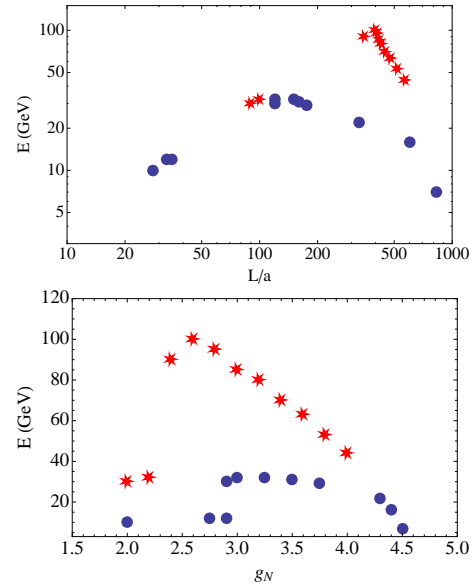


FIG. 6: Upper plot: The energy of the cluster E (GeV) versus the length of the string L/a . Lower plot: The energy of the cluster E (GeV) versus the “Newton coupling” g_N (GeV^{-2}). Points show the results of the simulations in setting $T_0 = 1$ GeV and size of the ball $s_T = 1.5a, 2a$, for circles and stars, respectively.

string ball of an average length of about 20 links, further reduction to $s_T = 1.0a$ (light orange histogram) shows that the most probable is the shortest configuration with 8 points (6 links), corresponding to an unexcited initial configuration. Yet even in this case, the population of the excited strings still show a long tale, with population up to 25 links (in this simulation), with a probability rate of about a percent. Inspection of those configurations shows that it is dominated by the excitation of one of the strings only, see lower part of Fig. 5.

B. Self-interaction included

Now we are ready to see how nonzero string self-interaction modifies the properties of the system. While increasing the corresponding parameter – “scalar Newton’s constant” g_N – we observe that above its critical value even the most basic features of the system change.

In Fig. 6 (upper figure) we show the calculated relation between the average string length L and its energy E . Each point is a run of about 10’000 iterations of the entire string updates after equilibration. While at small coupling E and L are simply proportional to each other, like for non-interacting strings described above, this behavior changes abruptly. As the negative self-interaction energy become important, the total energy E of the ball becomes *decreasing* with the string length L . In Fig. 6 (lower figure) we show more details of this behavior: this

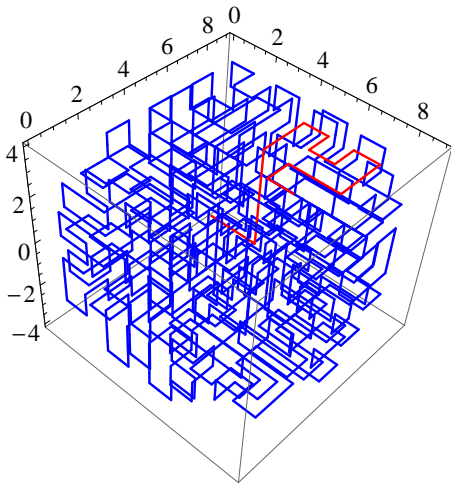


FIG. 7: (Color online) A typical configuration in the entropy-rich self-balanced string balls ensemble. Simulation parameters: $T_0 = 1 \text{ GeV}$, $s_T = 1.5a$, $g_N = 4.4 (\text{GeV}^{-2})$.

plot demonstrates how total energy E depends on the coupling value g_N . We find a jump at the critical coupling (for this setting) g_N^{c1} , which in a simulation looks like a first order transition, with double-maxima distributions in the energy and length. As is seen from the figure, the precise value of the coupling somewhat depends on the system size. At this coupling the jump in energy is always about a factor 3, and the jump in string length (or entropy) is even larger.

In this way we observe a new regime for our system, which we will call the “entropy-rich self-balanced string balls”. For a given fixed mass M we thus find that string balls may belong to two very distinct classes: (i) small near-random balls and (ii) large ones in which the string can be very long, but balances its tension by a comparable collective attraction. Discovery of this second regime is the main result of this paper.

Finally, there exists the second critical coupling, which found to be $g_N^{c2} \approx 4.5 \text{ GeV}^{-2}$, above which balancing the energy becomes impossible and simulations show immediate collapse of the system, in which the energy quickly falls to large negative values, clearly of no physical meaning.

Example of a corresponding configuration is shown in Fig. 7. Note that, in spite of a very large string length $L/a \sim 700$, the total energy is only $E \approx 17 \text{ GeV}$, as a result of the balancing between the mass and self-interaction. Note furthermore that mostly one string is excited, and that nearly all space inside the ball with $T > T_H$ is occupied. High entropy corresponds to a (astronomically) large number of shapes this string may have.

V. APPLICATIONS

A. Jet quenching during the mixed phase

Hard collisions, creating quark and gluon jets, provide an “X-ray tomography” of the excited fireball produced in heavy ion collisions. “Quenching” (absorption, modification) of such jets is one of the main diagnostic tools used to probe various phases of the hadronic matter appearing during the fireball expansion. The theory of jet quenching is rather involved, and the phenomenology is even more complicated, due to the time evolution of the fireball. For a recent summary see e.g. a report of the JET collaboration [17] and references therein. For our purposes it is enough to mention that the relevant matter properties are described by a single quantity,

$$\hat{q} = \frac{d\langle p_\perp^2 \rangle}{dl}, \quad (24)$$

characterizing increase of the mean squared momentum perpendicular to the direction of motion, per unit length.

According to an original assumption, this quantity is simply proportional to the entropy density s of the matter,

$$\frac{\hat{q}}{s} \approx \text{const}, \quad (25)$$

since both of them have the same mass dimension. Such a naive assumption is reasonable for the QGP phase, which is quasi-conformal and possesses only one scale – say T – of its own. But obviously there is no reason to extend this assumption to the mixed and hadronic phases, as their structure is quite different, especially in respect to the color field distribution affecting \hat{q} . The characteristic values used in current jet quenching models can be seen in Fig. 10 of [17]: for $T = T_c$ (the mixed phase) they range in the following interval

$$\left(\frac{\hat{q}}{T_c^3} \right)_{\min} \approx 1, \quad \left(\frac{\hat{q}}{T_c^3} \right)_{\max} \approx 6. \quad (26)$$

Note that the analysis in [17] is so far based only on the quenching strength itself: analysis of the quenching for jet paths with different azimuthal angles – or the so called $v_2 = \langle \cos(2\phi) \rangle$ at large p_t – is yet to be performed.

It has however been pointed out long ago [18] that large experimental values of v_2 are difficult to explain by any simple model of quenching, in particular, they were in a strong contradiction with the simplest assumption (25). One possible solution to this puzzle has been suggested few years ago in Ref. [6]: the v_2 data can be reproduced, if \hat{q} is significantly enhanced in the mixed phase. More recent data, especially from LHC, had shown that v_2 has, in fact, a rather strong p_t dependence and is decreasing with p_t of the observed hadron: so the issue seems to exist only for $p_t < 40 \text{ GeV}$ or so. Comparison of those data with various models and discussion can be found in Refs. [19, 20].

Here we want to point out that a natural explanation for the enhanced \hat{q} in the mixed phase can be provided by the strings. As far as we know, the “kicks” induced by the color electric field inside the QCD strings has been ignored in all jet quenching phenomenology: only the fields of “charges” (quarks and gluons in QGP, hadrons alternatively) were included, in the spherical Debye approximation. However, if the entire flux of the color-electric field is inside the QCD strings, there are no Coulomb fields of the charges and their Debye cloud. There are, in fact, two different reasons for it to be the case: (i) a generic string enhancement due to the Hagedorn phenomenon; and (ii) further enhancement energy due to the string self-interaction, the main subject of this paper. We will discuss below those two effects subsequently.

As we repeatedly emphasized already, in the mixed phase the strings are close to their Hagedorn temperature, so they get easily excited. Let us refer to their average length as \bar{L} , and to the string radius as r_s . The geometrical cross section of the jet-string interaction scales as their product: we will use $2\bar{L}r_s$.

More accurately, approximating the QCD cross-over transition by a first order transition, one defines the mixed phase as $T = T_c \approx 0.17 \text{ GeV}$ and variable energy (and/or entropy) density. The normalized energy density according to lattice calculations (now for the QCD with quarks, not just for gluodynamics, as in section III), ϵ/T^4 , ranges from 3 at $T = T_c$ to about 12 at $T = 1.2 T_c$, or $0.3 \dots \text{GeV/fm}^3$. Assuming that all this energy comes from a string, and dividing naively by the vacuum ($T=0$) string tension σ_T , one finds that inside each 1 fm^3 cube there is a string of length changing between $\bar{L}_{min} = 0.4$ and $\bar{L}_{max} = 1.4 \text{ fm}$, across the mixed phase.

Let us now estimate \hat{q} , by a simple classical argument. The mean square of the momentum kick we write as

$$\langle p_\perp^2 \rangle \approx (gEr_s)^2, \quad (27)$$

which is a color force, gE , times the time it acts while the jet is traversing the string, (some coefficient of the order one times) r_s . This combination of the field strength and the radius can be directly obtained from the following consideration: the string tension, i.e. the energy per length, is that of the field inside the string plus the energy of the “coil” (the magnetic current holding the field). The former one is $(E^2/2)\pi r_s^2$: and the latter should be comparable. Assuming it is the same, and eliminating $1/2$ we get $\sigma_T = \pi r_s^2 E^2$ from which it follows that

$$\langle p_\perp^2 \rangle \approx 4\alpha_s \sigma_T \quad (28)$$

The geometric probability for a jet to cross the string is $\frac{2}{3} \frac{\bar{L}r_s}{\text{fm}^2}$ over each fm longitudinally. Here $(2/3)$ excludes string segments along the jet, in which the kick is longitudinal. So,

$$\hat{q} \approx \frac{16}{3} \alpha_s \sigma_T \frac{\bar{L}r_s}{\text{fm}^3}. \quad (29)$$

We still need to know the string radius and, fortunately, its value and the string profile have been extensively studied on the lattice. Furthermore, in the so-called dual Abelian model the QCD strings – flux tubes – are the well known Abrikosov vortex solutions. Numerical data and the dual theory do, in fact, agree quite well: see, in particular, a review by Bali [21], from which we borrow a fit to the lattice data, by the profile function

$$E(x) = \frac{\Phi_e}{2\pi r_s^2} K_0(x/r_s) \quad (30)$$

with K_0 being the Bessel function. The main point here is the value of the string radius $r_s = 1/(1.3 \text{ GeV}) = 0.15 \text{ fm}$. The normalization parameter is $\Phi_e = 1.44$.

Now all parameters in the \hat{q} expression above are fixed and we can evaluate \hat{q} numerically. With $\alpha_s = 1/2$ one finds the range across the mixed phase to be

$$\hat{q}_{min} = 0.028, \quad \hat{q}_{max} = 0.10 \left(\frac{\text{GeV}^2}{\text{fm}} \right). \quad (31)$$

Comparing these estimates with the values used in the phenomenological models by the JET collaboration (26) mentioned in the beginning of this subsection. Putting them in the same absolute units one finds those to be

$$\hat{q}_{min} = 0.025, \quad \hat{q}_{max} = 0.15 \left(\frac{\text{GeV}^2}{\text{fm}} \right), \quad (32)$$

which is in a good correspondence with our estimates.

This agreement does not, of course, mean that either the estimate or empirical inputs used are, in fact, correct. Recall that the JET collaboration’s analysis is done for the hadron $p_t \sim 10 \text{ GeV}$, well inside the region, in which the large v_2 puzzle remains unresolved. If these data are to be included in their analysis, the values would go up.

From the theory side, the presented estimate looks suspicious, because it does not include the second enhancement effect, that is due to the string self-interaction. Indeed, above we assumed the energy of the string to be just linear in length due to its (vacuum) tension, i.e. $L\sigma_T$. But, as we demonstrated in the upper Fig. 6, the “entropy-rich branch” of the string balls has a different relation between the total energy and the string length L : self-interaction can compensate a large fraction of the energy. For the same total ball energy its string length L can, in fact, be up to an order of magnitude larger, reaching, perhaps, $\hat{q} \sim 1 \text{ GeV}^2/\text{fm}$ magnitude range, which is usually associated with the QGP phase. Since the string inside still contains the same electric flux etc, it means that \hat{q} can be enhanced by this mechanism by about an *order of magnitude*. (Another glance at our extreme string-ball configurations shown in Fig. 7 may be needed at this point, for most skeptical readers.) If this is the case, the mechanism behind large v_2 will be explained!

Of course, it is just a possibility at this point: as we had shown, it happens provided the self interaction parameter happen to be of the right magnitude. Unfortunately, we

don't really know what is its real-world value. (Again, we only see that what is needed is several times smaller than sigma coupling to the nucleons, which binds them into nuclei.)

B. Angular correlations

The first difference between the typical and high-multiplicity pp and pA collisions first discovered was the so-called “ridge” correlations. Soon LHC experiments had shown that it is, in fact, complemented by an “anti-ridge” out of plane, with the second $n = 2$ harmonics

$$v_n = \langle \cos(n\phi) \rangle \quad (33)$$

in the azimuthal angle. Also (smaller) third harmonics were observed: both fit well to the hydrodynamic systematics from AA collisions.

Hydrodynamics explains these azimuthal moments by relatively small deformations of basically axially symmetric ($n = 0$) flow, known as the radial flow. It has been predicted by Shuryak and Zahed [22] that the magnitude of such radial flow in pp should be larger than in pA , which is larger than ever observed in AA collisions. Using spectra of identified secondaries it has indeed been observed, by CMS and ALICE collaborations [24, 25], see also [23]. So, supporting the hydrodynamical “explosive” interpretation of high multiplicity pp/pA events is now becoming a mainstream activity.

The analysis just mentioned deals with the average values of angular correlation parameters v_n : one may also address the question of a *distribution* over them. Not going into detailed discussions, we find it sufficient to view hydrodynamics simply by classical means, translating the initial spatial deformation of the system

$$\epsilon_n = \frac{\int d^2 r_\perp \cos(n\phi) r_\perp^n (dN/d^2 r_\perp)}{\int d^2 r_\perp r_\perp^n (dN/d^2 r_\perp)} \quad (34)$$

in the azimuthal angle ϕ into the observable flow parameters v_n , via certain calculable linear response coefficients

$$\frac{v_n}{\epsilon_n} = f_n^{hydro}. \quad (35)$$

Studies of the AA and pA collisions in the Glauber model provides not only the average means but also distributions $P(\epsilon_n)$. Observed distributions over v_n are well described by those, so linear response relation between them is believed to be valid on the event-by-event basis.

In “event generators”, describing the initial state, one determines ϵ_n for each event in the ensemble, obtaining certain distribution of their values. The average over those will be denoted by the angular brackets $\langle \dots \rangle$. The experimental measurements involving different numbers of particles are traditionally encoded in the following

combinations of moments of the distribution (see derivation and discussion in e.g. [26]),

$$(\epsilon_n \{2\})^2 = \langle \epsilon_n^2 \rangle, \quad (36)$$

$$(\epsilon_n \{4\})^4 = 2\langle \epsilon_n^2 \rangle^2 - \langle \epsilon_n^4 \rangle, \quad (37)$$

$$(\epsilon_n \{6\})^6 = \frac{1}{4} [\langle \epsilon_n^6 \rangle - 9\langle \epsilon_n^2 \rangle \langle \epsilon_n^4 \rangle + 12\langle \epsilon_n^2 \rangle^3], \quad (38)$$

$$(\epsilon_n \{8\})^8 = \frac{1}{33} [-\langle \epsilon_n^8 \rangle + 16\langle \epsilon_n^6 \rangle \langle \epsilon_n^2 \rangle + \quad (39)$$

$$18\langle \epsilon_n^4 \rangle^2 - 144\langle \epsilon_n^4 \rangle \langle \epsilon_n^2 \rangle^2 + 144\langle \epsilon_n^2 \rangle^4]. \quad (40)$$

Assuming the linear response (35) holds on the event-by-event basis, one finds that it will also hold for v_n distributions, as all terms in the formulae above scale in the same way.

The corresponding combinations are called similarly $v_n \{m\}$: the parameter m in this case has a meaning of the number of secondaries used to produce this correlation. We will make two brief comments on those parameters. One is that “classical” (=non-fluctuating) distribution $P(v_n) = \delta(v_n - V_n^0)$ makes all of them being the same, namely V_n^0 . However, the empirical pattern observed in AA and pA collisions is a bit different, i.e.

$$v_2 \{2\} > v_2 \{4\} \approx v_2 \{6\} \approx v_2 \{8\}, \quad (41)$$

which implies that there exist certain 2-body correlations absent for higher $m > 2$. We will not go into the further discussion present in the literature of what exactly does it tell us about the underlying physics.

While fluctuations in the location of the nucleons are believed to be the main cause of angular shape fluctuations – thus, Glauber treatment – of AA and pA collisions, so far there is no consensus or even studies of what can be the source of those in pp collisions. So far, studies of the proton structure in terms of parton distribution functions (pdf's) were restricted to their longitudinal momenta only, and also to minimal biased (mean) protons. Only qualitative speculations on the transverse shapes of the high multiplicity events were proposed: for example, one suggestion made by Bjorken et al. [27] is that such events come from the collisions of near-parallel strings. If so, one may expect very large initial deformations $\epsilon_2 \approx 1$, but $\epsilon_3 \approx 0$.

Since ensemble of string balls generated by our partition function have various angular shapes, and since some of those may be driven by the underlying explosive hydro (as explained in the Introduction), some study of their angular fluctuations and statistical ensembles involved are of a certain interest. Note, that because points on our lattices are connected by the string, their statistics may be different from the ensembles of mutually independent points (the “wounded nucleons”) in the Glauber approach.

In Fig.8 we show the (non-normalized) distributions $P(\epsilon_2), P(\epsilon_3)$ for three ensembles of string balls. The first observation following from those distributions is that the larger are the balls, the smaller are their deformations.

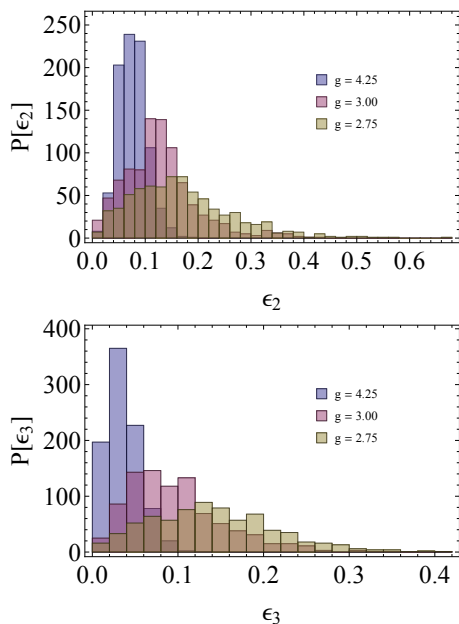


FIG. 8: The distributions over ϵ_2 and ϵ_3 (upper and lower plots), for several values of the “Newton coupling” g_N (GeV^{-2}).

This is, of course, expected on the general grounds: note, however, that for our largest ball the peak values are at $\epsilon_2 \approx 0.06$, $\epsilon_3 \approx 0.03$, which are quite small, in the ballpark of those in AA collisions with heavy nuclei. The second qualitative feature of the distributions is that they are quite far from the delta function or narrow Gaussian: in fact, there exists a rather long tail toward large values of deformations. This is a consequence of near-critical fluctuations of strings on the ball surface, allowing for a rather large string suddenly protruded outward (as e.g. is the case for magnetic flux tubes on the Sun). The third feature is more developed tails for ϵ_3 than for ϵ_2 : in difference to Glauber ensembles, which show rather similar fluctuations in all ϵ_n up to high n , because of the point-like angular contributions of the “wounded nucleons”. Our strings are not point-like objects, and this difference, indeed, reveals itself.

The specific values of the parameters (for the $g_N = 4.25 \text{ GeV}^{-2}$, the largest used) are

$$\epsilon_2\{2\} = 0.0759, \quad \epsilon_2\{4\} = 0.0621, \quad (42)$$

$$\epsilon_2\{6\} = 0.0636, \quad \epsilon_2\{8\} = 0.0635, \quad (43)$$

which indeed fits the pattern (41), whatever it may mean. However, some of the smaller balls have ensembles which fluctuate so much as they violate the indicated pattern, sometimes to the extent that the r.h.s. of (37) become negative, or $\epsilon_2\{4\}$ complex.

We are looking forward toward experimental measurements of those distributions/parameters in the high multiplicity pp collisions.

VI. SUMMARY AND OUTLOOK

A. Summary

In this paper we have formulated a new lattice string model, reminiscent of the Wilson’s strong coupling expansion idea. Its lattice spacing a is not, however, an artificial discretization parameter, going to zero at the end of the calculation. Instead, it is chosen to imitate a physical density of state of the QCD string.

Our setting also differs from the lattice gauge theories in the infrared. Instead of a certain periodic box, we have chosen to have an infinite lattice, but with a space-dependent temperature $T(x)$. The resulting “string balls” of a certain pre-determined size are expected to model excited systems created both at the mixed phase of the heavy ion collisions and also in some high multiplicity pp scattering events.

The main physics of the model is related to the string self-interaction. While this has been discussed in the fundamental string theory context – with the self-interaction due to gravity, leading sometimes to the creation of black holes – we believed it has never been discussed for QCD strings. In particular, various event generators widely used by experiments, all follow the Lund model treatment of strings, i.e. they are simply straight in shape and non-interacting.

The main result of the simulation we made in the model’s setting is a discovery of a new class of string balls, in which energy of the string is balanced by the self-interaction. Such objects may form a relatively modest mass with rather large entropy, and thus provide the closest QCD-based objects to the gravitational black holes.

Obviously, those objects are of significant theoretical interest. After all, fundamental string theory is still remote from experiments, by many orders of magnitude, and small-mass gravitational black holes are unavailable. QCD systems we discuss are, on the other hand, produced in the numbers of billions at hadronic and heavy ion collisions, providing statistically interesting samples of many categories of the final states. Strong collective fields, if exist in some of those systems, must in particular give rise to the massive pair production at their edge, analogous in spirit to the Hawking radiation. Large entropy and its possible upper bounds should be in a way analogous to the Bekenstein entropy of the black holes. Impossibility of climbing out of a deep potential well, gravitational or not, causes the famous information paradox, which is hotly debated in literature for a long time.

Furthermore, as we comment on a bit in the next subsection, various versions of AdS/QCD models bring QCD string balls and gravitational black holes together in a rather direct sense via the holography.

String systems with large entropy are also of significant experimental interest. As entropy never decreases, those must be related to a larger entropy at later hadronization stages, and thus can be detected via unusually large

multiplicities. As we commented on in the introduction, LHC experimental triggers were able to select statistically significant ensembles of such events in pp, pA collisions, and demonstrate that their behavior is quite different from the minimal biased or “usual” events. In the previous section we, in particular, argued that the high multiplicity events correspond to the high-entropy string balls: one consequence of it is a relatively small angular deformation ϵ_2 .

B. Outlook

As we already emphasized in the Introduction, one rather straightforward extension of this work can be performed within the holographic AdS/QCD framework. This implies that one of the three spatial dimensions of the model is interpreted as a holographic curved coordinate z . The bulk fields, dilation and graviton, are massless, but their motion in z gets quantized due to a confining wall, and the corresponding 5-momenta play the role of the sigma mass in our Yukawa potential.

Perhaps, more demanding in terms of work would be its extension from the early time – when string configurations are quantum fluctuations in transverse coordinates described by the semiclassical “effective temperature” – to later times. An individual QCD string, from being transverse at the initial collision time $t = 0$, gets stretched longitudinally and then breaks, first into clus-

ters and then to final lightest hadrons reaching the detectors. Stretching of a self-interacting string ball is an interesting problem, not yet studied.

The holographic strings cannot be broken – as considered AdS/QCD models lack fundamental charges in the bulk – but simply fall to the “bottom of space”, i.e. off the boundary, influencing holographically the stress tensor and other operators on the boundary. A holographic string ball, if heavy enough, makes a full-scale bulk black hole, with all its prerequisites – Hawking-Bekenstein temperature and entropy, as well as gravitational fall away from the boundary, dual to hydro explosion.

We hope to be able to report on the progress along some of these lines elsewhere.

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